DIAGNOSTICS OF CONVECTIVE PROCESSES IN THE BOUNDARY LAYER OF A LIQUID BY THE LASER-REFRACTOGRAPHY METHOD

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A mew method of nondisturbing remote diagnostics, i.e., laser refractography, has been used for experimental investigation of convective microflows. The theoretical foundations of the method and the principles of construction of a measuring system have been presented. The dynamics of the temperature distribution in the boundary layer at a cooled or heated sphere in water in the presence of free convection has been diagnosed quantitatively. The results of the experiment have been compared to the results of computer simulation based on the FLUENT software package. The advantages of using laser refractography for diagnostics of substantially nonstationary processes have been demonstrated.

Keywords: laser refractography, structured laser emission, diffraction optical elements, convective flows.

Introduction. The topicality of experimental diagnostics of convective flows is conditioned by the wide range of natural phenomena and technical applications in which convection is of substantial importance [1]. Systems for air conditioning and cooling (including that by liquid carriers) which are used in today's electronics equipment, gas-turbine plants, chemical-engineering apparatuses, etc. are the most important in practical terms. Furthermore, in connection with the rapid recent development of micro- and nanotechnologies, it has become pressing to develop methods of quantitative diagnostics and visualization of microflows, including convection in boundary wall and surface layers of a liquid. When convective flows and temperature distributions in the boundary layer at the surface of a hot or cooled body in a liquid are investigated experimentally, it is often fundamental to use nondisturbing remote methods of diagnostics with a high resolution of the boundary-layer structure. All these conditions are satisfied by laser techniques [2–11] making it possible to carry out highly accurate measurements of the velocity and temperature field of the flow in microlayers of liquids and gases and to visualize convective flows.

In the present work, we use the laser-refractography method [2, 12, 13] for investigation of the boundary layer with a temperature gradient at a spherical surface. This method is based on probing of the medium under study with structured laser emission (SLE), recording of the emission refracted in the heterogeneous medium using digital video cameras, and computer processing of refraction images to reconstruct the heterogeneity structure.

1. Principles of Laser Refractography. In laser refractography, one uses SLE formed on the basis of special optical elements directly at the emitter outlet. This method of formation of SLE makes it possible to preserve its high coherence and to ensure low divergence of the beams, which enables one to use representations of geometrical optics for SLE description. Within the framework of geometrical optics, the SLE model can be represented by the families of rays forming surfaces as a discrete set of planes, nested cylinders, cones, and others.

Structured laser emission transmitted by the heterogeneity is projected onto the screen in the observation plane, forming the so-called 2D refractogram. The high SLE intensity allows observation of 3D refractograms, i.e., surfaces formed by refracting geometrical-optics rays, in the scattered emission. Figure 1 shows 2D refractograms for different forms of SLE and typical heterogeneities (heated bodies in water). Picture 8 in this figure is an example of a 3D refractogram.

Clearly, the initial structure of the beam, which is described by a number of information parameters, changes with its refraction in the optical heterogeneity, which enables one to carry out quantitative diagnostics and visualization of the heterogeneity from the refractograms obtained experimentally.
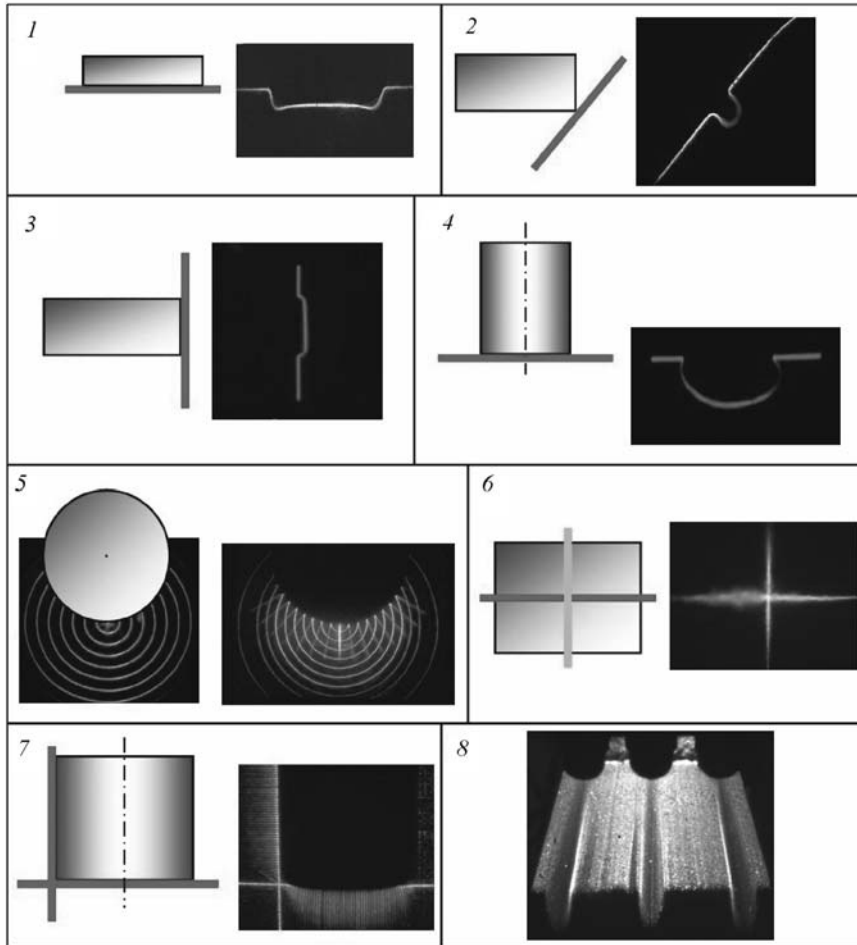
Fig. 1. Library of laser refractograms: 1) bottom of a parallelepiped; 2) angle of a parallelepiped; 3) lateral wall of a parallelepiped; 4) bottom of a cylinder; 5) heated sphere; 6) mixing of liquids; 7) lateral wall and bottom of a cylinder; 8) three heated spheres (in scattered light).

Diffraction optical elements which represent a transmitting or reflecting plate with a thin phase microrelief calculated within the framework of diffraction theory are the element base of the Gaussian laser-to-structured beam conversion. The development of microelectronics and laser engineering has made practicable the problem of creation of diffraction optical elements with a complex zone profile and unique characteristics unattainable within the framework of traditional optics. Diffraction optical elements focusing laser emission to thin lines or small space regions, which corresponds to a structured beam with intensity modulation [14], are the most likely candidates for laser refractography. In this case the structural elements of the beam are directly visualized in its cross section and the refractograms are of a contour graphic character (Fig. 1), which actually gave birth to the term "refractography."

The use of beams with such a structure in laser refractography substantially extends the capabilities of traditional laser gradient methods. First, the appropriate selection of the configuration of the elements of the beam structure makes it possible to adapt measurements to a certain type of heterogeneities, e.g., to investigate boundary microlayers at variously shaped surfaces. Second, the multiplicity of the structural elements of the beam and their spatial length allow simultaneous measurements in the selected regions of the medium under study, which is significant in diagnosing dynamic heterogeneity, e.g., nonstationary convective flows. Third, the basic informative parameter in quantitative diagnostics is the displacement of the structural element of the beam because of the refraction in the heterogeneity. Measurement of this displacement turns out to be possible owing to the identification of the elements, since the discrete and regular character of the beam structure enables one to determine what structural element in the initial beam corresponds to one refractogrammatic element or another.
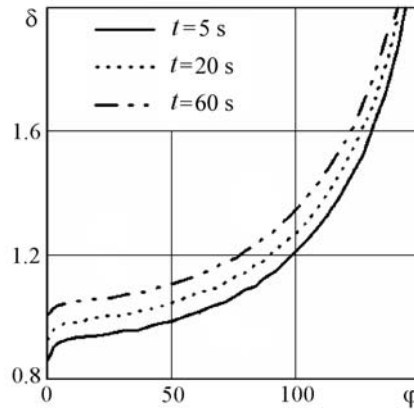
Fig. 2. Thicknesses of the boundary layer δ along the sphere's surface vs. angle with a vertical axis φ at different instants of time.

Thus, the use of structured beams in laser refractography makes it possible to actually carry out multichannel probing of the medium with spatial separation of the identified channels and with the possibility of adapting spatially to the shape of the investigated heterogeneity. The enumerated features of laser refractography enable us to say that this method makes it possible not only to visualize optical heterogeneities but to diagnose them quantitatively as well.

2. Numerical Simulation of the Boundary Temperature Layer at the Surface of a Heated Sphere. The licensed FLUENT ver. 6.3.26 software package [15] is used for numerical simulation. The problem of formation of the temperature boundary layer is considered with the example of convective flow occurring in the vicinity of a quiescent steel sphere which is immersed in a liquid (the liquid and the sphere initially have different temperatures) and is in the gravitational field. The computational domain includes the steel sphere of diameter 50 mm heated to a prescribed temperature and placed in a water-filled cylindrical vessel of diameter 125 mm. Solution is based on integration, by the finite-volume method, of unsteady equations of motion of a viscous heat-conducting liquid and heat-conduction equations [16, 17] for a sphere, which are written in axisymmetric form. The tetragonal difference grid used includes about 52,000 cells.

In what follows, we give results of numerical simulation of flow occurring near the walls of a hot sphere which is heated to a temperature of 95°C and is placed in cold water with a temperature of 20°C. An analysis of the data obtained in the calculation has shown that the heated water ascends along the surface of the sphere to its upper point and then upward along the axis of symmetry, where it spreads at the sphere's surface. In the initial period of time, a small circulation zone occurs on the axis of symmetry near the upper point of the sphere; in this zone the liquid constantly remains colder than that ascending around it. By a time of about 3 s, the water inside this zone is heated to the temperature of the liquid passing by, and this circulation zone itself becomes vanishingly small. It is assumed that such a zone near the axis of symmetry exists at all times; however the possibility of finding it is limited by the resolving power of the difference grid.

Figure 2 plots the thicknesses of the boundary layer δ along the sphere's surface versus the angle with a vertical axis φ at different instants of time [17]. The distance from the spherical surface to the liquid layer in which the water temperature becomes equal to 300 K is selected as the boundary of the boundary layer.

When the sphere cools down, the boundary layer formed near it is exposed to two opposite effects. On the one hand, heat continuously propagates from the heated part of the liquid to the colder one, which causes the boundary-layer thickness to monotonically grow. On the other hand, the temperature of the sphere's surface diminishes with time. This leads to a decrease in the heat flux from the sphere to the liquid and to a reduction in the temperature gradient in the liquid itself near the sphere.

3. Investigation of Natural Convection by the Laser-Refractography Method. *3.1. Procedure of calculation of refractograms.* To develop a procedure for comparison of the experimental and calculated dependences of temperature in the boundary liquid layer we construct calculated refractograms of a plane SLE (laser plane) in a spherically layered medium with a refractive index dependent on temperature in accordance with the approximation relation
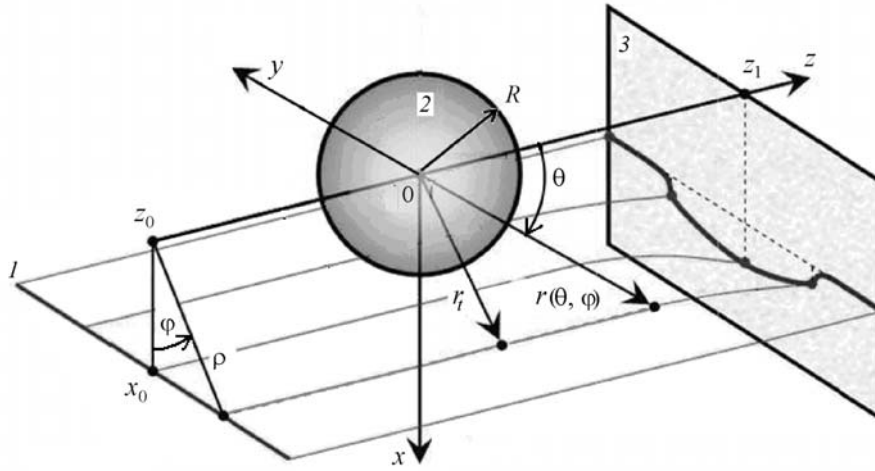
Fig. 3. Geometry of the problem: 1) laser plane; 2) heated sphere; 3) observation plane.

$$n\,(T) = 1.3328 - 0.000051T - 0.0000011T^2 , \tag{1}$$

which has been obtained for water from the dispersion formula and the data of [18] for laser emission with a wavelength $\lambda = 0.6328$ μm.

The geometry of the problem is illustrated by Fig. 3. The laser plane 1, which is perpendicular to the $0x$ axis, extends along the $0z$ axis. The origin of coordinates is made to agree with the center of a heated sphere 2. The projection of the laser plane is observed in plane 3, which is perpendicular to the $0z$ axis and is at a distance z_1 from the origin of coordinates.

The relationship between the Cartesian and spherical coordinates is prescribed by the relations

$$x = r \sin \theta \cos \varphi , \quad y = r \sin \theta \sin \varphi , \quad z = r \cos \theta . \tag{2}$$

When $z = z_0$ the equation of the laser plane has the form $x = x_0$. The impact parameter of the ray lying in this plane is determined by the parameter φ and is equal to

$$\rho\,(\varphi) = \frac{x_0}{\cos \varphi} . \tag{3}$$

In the case of spherical symmetry this ray remains in the plane prescribed by the parameter φ and its trajectory is determined by the dependence $r(\theta, \varphi)$. The ray trajectories are calculated from the relations describing the propagation of the ray in a spherically layered heterogeneity [19].

The radial coordinate of the ray at entry into the medium (when $z = z_0$) is equal to

$$r_0\,(\varphi) = \sqrt{\rho^2\,(\varphi) + z_0^2} . \tag{4}$$

The angle θ_0 characterizes the direction of the ray at entry into the medium:

$$\theta_0\,(\varphi) = \frac{\pi}{2} + \arctan \frac{z_0}{\rho\,(\varphi)} . \tag{5}$$

The angle $\theta = \theta_t$ corresponding to the radial coordinate $r_t(\varphi)$ of the turning point is equal to
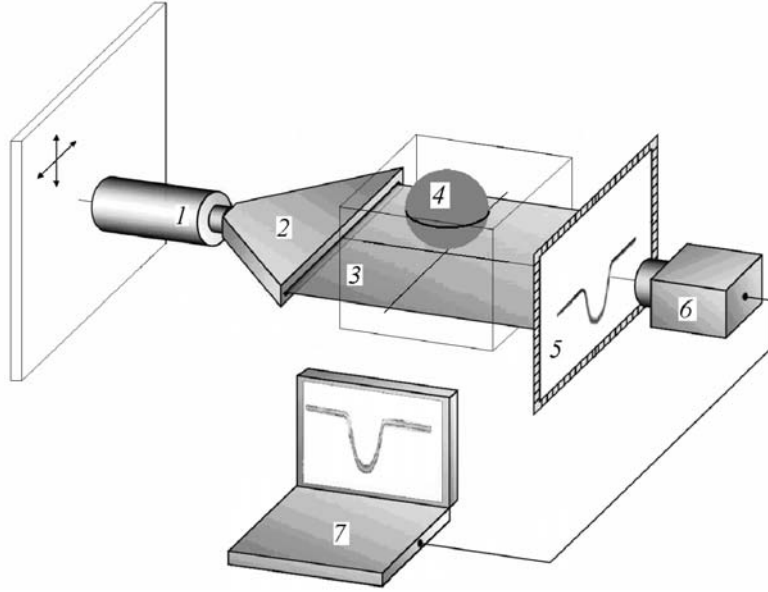
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Fig. 4. Block diagram of the laser refractographic system: 1) laser; 2) optical system of formation of the laser plane; 3) laser plane; 4) heated sphere; 5) matte screen; 6) digital video camera; 7) computer.

$$\theta_t (\varphi) = \theta_0 (\varphi) + \int\limits_{r_0(\varphi)}^{r_t (\varphi)} \frac{n_0 \rho (\varphi)\, dr}{r \sqrt{n^2 (r)\, r^2 - n_0^2 \rho^2 (\varphi)}} .$$ (6)

The ray equations before the turning point and after it have the form

$$\theta (r, \varphi) = \theta_0 (\varphi) + \int\limits_{r_0(\varphi)}^{r} \frac{n_0 \rho (\varphi)\, dr}{r \sqrt{n^2 (r)\, r^2 - n_0^2 \rho^2 (\varphi)}} ,$$ (7)

$$\theta (r, \varphi) = \theta_t (\varphi) + \int\limits_{r}^{r_t(\varphi)} \frac{n_0 \rho (\varphi)\, dr}{r \sqrt{n^2 (r)\, r^2 - n_0^2 \rho (\varphi)}} .$$ (8)

Formulas (7) and (8) form the basis for calculating the refraction of the laser plane in the radial heterogeneity. The angle φ is the parameter prescribing an arbitrary ray in a laser plane, which makes it possible to describe the entire family of rays belonging to this plane. On the screen at a distance z_1, the radial coordinate $r(z_1, \varphi)$ is determined from the equation

$$r \cos \theta (r, \varphi) = z_1 ,$$ (9)

and the coordinates of the projection of the plane on the screen are determined as

$$x (z_1, \varphi) = r (z_1, \varphi) \sin \theta (r (z_1, \varphi), \varphi) \cos \varphi ,$$

$$y (z_1, \varphi) = r (z_1, \varphi) \sin \theta (r (z_1, \varphi), \varphi) \sin \varphi .$$ (10)

Relations (10) determine the structure of the refractogram for the prescribed spherical heterogeneity. Typical refractograms are given in [2]; part of them are presented in Fig. 1.
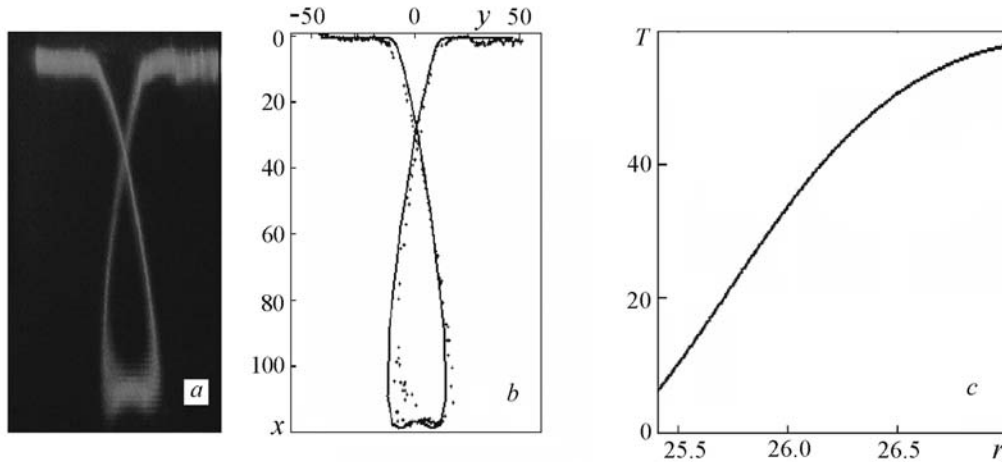
Fig. 5. Reconstruction of the temperature profile for a cold sphere: a) experimental refractogram; b) comparison of the experimental and calculated refractograms; c) reconstructed dependence of the temperature in the boundary layer. T, $^{\circ}$C; r, mm.

3.2. Experimental setup. The laser refractographic method of diagnostics and visualization of the optically heterogeneous transparent medium under study is implemented by a series of the following operations [2]: probing of the medium with an astigmatic laser beam (laser plane), recording of the refractogram on a matte screen using a digital video camera, and processing of the resulting refractograms using an applied software package to obtain the temperature distribution in the boundary layer.

For the optical heterogeneity to be investigated by the laser-refractography method, the measuring system must contain the following basic elements: a laser emitter, an optical unit of formation of the laser plane, a system for positioning of the laser plane, a diffusely scattering translucent screen, a digital camera, and a personal computer. The object of investigation is placed between the optical system of formation of the laser plane and the matte screen. The laser-plane refractogram can be visualized on the reflecting or transparent matte screen and be recorded by the digital camera. Next, computer processing of the refractogram can give information on the gradient of the refractive index of the medium in the domain under study and hence on the distribution of the gradient of the physical field in it, from which we can reconstruct the corresponding distribution function of this field.

Figure 4 shows the block diagram of a laser refractographic system based on plane-structured laser emission. It consists of laser 1 with an optical system of formation of a laser plane 2. The laser and the optical system are installed on a traversing probe making it possible to move the laser plane in two directions in space. The laser plane 3 can be located on different sides of a heated steel sphere 4 placed in a water-filled cell. The refractogram is visualized on a matte screen 5, is recorded by a digital video camera 6, and is processed on computer 7.

3.3. Algorithm of reconstruction of the temperature profile. The discrete and regular character of SLE ensures the possibility of diagnosing the medium quantitatively from the experimentally obtained refractograms. Quantitative diagnostics implies solution of the inverse problem; it involves determination of the parameters of the heterogeneity (when its parametric model is prescribed) or reconstruction of its profile in the form of a finite set of numbers. Diagnostics of an optically heterogeneous liquid around a heated or cooled sphere is in essence a problem of tomography, to solve which one generally uses the integral Radon transform. For axisymmetric heterogeneities, the Radon transform is reduced to the Abelian transform having analytical inversion. Problems arising when the integral Abelian equation is used are associated with the incorrectness of the inversion problem, since it is necessary to differentiate noised experimental measurements and to overcome a singularity in the integrand. Solution of the primal problem of refraction of the family of rays forming SLE and calculation of the corresponding refractograms can be an alternative to such an approach. Subsequent computer processing of the experimental refractograms and their comparison to the calculated ones make it possible to select a law of variation in the refractive index that ensures the agreement of the calculated and experimental data. The limiting error in measuring the refractive-index gradient is mainly determined by diffraction effects and can be diminished to a large extent by methods of special processing of the refractograms.
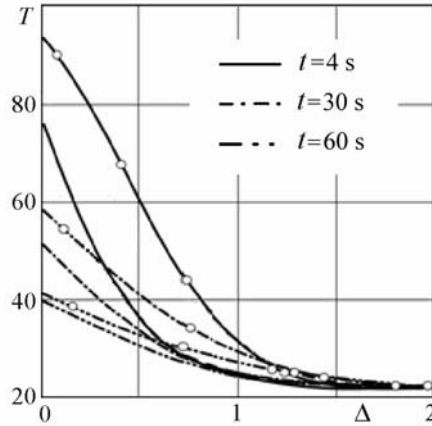
Fig. 6. Change in the temperature in the boundary layer in the process of cooling of a hot sphere in water for different instants of time. Dots, experiment; curves without dots, calculation. T, °C; Δ, mm.

Let us consider the algorithm of quantitative diagnostics of the heterogeneity, using as an example an analysis of refractograms for a spherical temperature layer [2]. It consists of the following steps. First, the two-dimensional refractogram of the laser plane is recorded using a digital camera, next its special computer processing minimizing diffraction effects is carried out; the digitized experimental refractogram is compared to the set of library refractograms calculated for a given setup and typical profiles of the temperature layer; the parameters of the calculated refractogram that is in best agreement with the experimental one are selected from the criterion of minimization of the standard deviation; the profile corresponding to the selected calculated refractogram is chosen as the measured temperature profile. In solving thermal problems for the boundary layer, one usually selects the temperature dependence in the form of an exponential function. However, to extend the capabilities of the computational method we select a parametric model of the form [2]

$$T(r) = T_0 + \Delta T \exp\left[-\frac{(r - R + \Delta R)^2}{a^2} \right]. \tag{11}$$

The parameter T_0 is determined by the temperature of the liquid at a fairly large distance from the sphere, the value of $T(R)$ is equal to the temperature on the sphere's surface, and the ratio $\Delta T / a$ corresponds to the gradient of the temperature field in the boundary layer of thickness a. The value of the temperature gradient at $r = R$ is determined by the ΔR shift. Figure 5 shows the recorded and processed refractograms and the corresponding radial temperature distributions in the boundary layer at a sphere cooled down to 7°C and placed in water with a temperature of 65°C.

4. Comparison of Experimental and Calculated Results. A comparison of the experimental results and calculation based on the FLUENT package reveals good agreement; the disagreement does not exceed 10% and may be due to the incomplete correspondence of the model used for calculation in the applied software package to experiment.

The radial temperature distributions in the boundary layer $T(r)$ for different instants of time corresponding to the instants of recording of refractograms [17] are summarized on one plot in Fig. 6. This figure illustrates the process of cooling-off of the sphere from the viewpoint of both the change in its surface temperature (sphere radius $R = 25.4$ mm) and the qualitative and quantitative change in the boundary-layer profile with time. Experimental curves for the point $\varphi = 0$ are dotted. Calculated curves are constructed for $\varphi = 10°$, i.e., beyond the zone of singularities of the temperature distribution.

The results of thermophysical calculations of the temperature in the boundary layer for the quasistationary process of convection based on the FLUENT applied software package are in good agreement with the exponential distributions $T(r)$ for instants of time of 30 and 60 s, but they disagree with the experimental curves for instants of 4 and 10 s; this is attributable to the fact that the process of convection is substantially nonstationary at first instants.

Furthermore, it has been found experimentally than an unstable ascending flow is formed at the upper surface of the heated sphere at the initial instant of time for the selected parameters, whereas the calculations give a stable

flow pattern by virtue of the axisymmetric formulation of the problem. Thus, the laser-refractography method is a fine instrument for investigating nonstationary processes in microlayers, which makes it possible to reveal new effects not describable by the classical methods of calculation of natural convection. It is of interest to further investigate the relationship between the parameters of the layer profile and its change and the physical characteristics of the surface and its microstructure.

Conclusions. Laser refractography can be used for investigation of nonstationary thermal processes in a liquid, including natural convection in a liquid at heated or cooled bodies, and for quantitative diagnostics of the temperature fields in the boundary layer with cooling and heating techniques. Furthermore, owing to the possibility of creating narrow probing beams the laser-refractography method is adapted to investigation of the processes in microstructures.

The procedure of investigation by the laser-refractography method ensures remoteness, virtual noninertia, and the possibility of nondisturbing measurements and measurements in microchannels. Fundamentally new features of the method are adaptation to the shape of the surface under study, the possibility of obtaining a 3D informative picture, simultaneity of measurement in different spatial domains (which is significant for investigation of nonstationary processes), quantitative diagnostics, and the possibility of visualizing directly and informatively in the process of monitoring.

A comparison of the results of numerical calculation and experiment shows that current numerical methods make it possible to diagnose heat fluxes between media and the rate of cooling of a body with a sufficient degree of accuracy. Computer simulation describes correctly in the main the structure of quasistationary convective flow in the vicinity of a sphere. Among the drawbacks of numerical methods are great resource-intensity and duration of such calculations even on high-power computers. The disagreements with calculation (presence of the singularity of the temperature distribution at the lower surface of the sphere along the axis of symmetry and the occurrence of unstable flow at the upper surface of the sphere) which are observed in the experiment call for further investigation.

Therefore, it seems expedient to continue investigations aimed at clarifying the conditions of existence of the singularity of the temperature distribution in the boundary layer on other models and at finding it, where possible, in experiment.

NOTATION

a, parameter of the temperature-field model, mm; n, refractive index of the medium; R, radius of the sphere, mm; ΔR, parameter of the temperature-field model, mm; r, spherical coordinate, mm; r_t, coordinate of a point in a heterogeneous medium, mm; r_0, coordinate corresponding to the point of entry of the ray belonging to the laser plane into the heterogeneous medium, mm; t, time, s; T, temperature, $^{\mathrm{o}}$C; $T(r)$, temperature of the liquid at a distance r from the center of the sphere, $^{\mathrm{o}}$C; T_0 and ΔT, parameters of the temperature-field model, $^{\mathrm{o}}$C; x, y, z, Cartesian coordinates; x_0, z_0, coordinates corresponding to the point of entry of the ray which belongs to the laser plane into the heterogeneous medium, mm; z_1, coordinate of the plane of location of the screen, mm; δ, boundary-layer thickness, mm; Δ, distance reckoned along the radial coordinate from the sphere's surface, mm; θ, spherical coordinate; θ_0, coordinate corresponding to the point of entry of the ray which belongs to the laser plane into the heterogeneous medium, deg; θ_t, coordinate of the turning point of the ray in the heterogeneous medium, deg; λ, laser-emission wavelength, μm; ρ, impact parameter of the ray, deg; φ, spherical coordinate, deg.

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